The Effective QCD Lagrangian and Renormalization Group Approach.

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Abstract

The perturbative part of the effective QCD Lagrangian was constructed by the renormalization group approach in the 4-loop approximation. It was extrapolated in the non-perturbative region under the assumption of the existence of gluon condensate at $(F^j_{\mu\nu})^2 = F_0^2$. The best result is $F_0^2 \approx 0.4$ GeV^4.

The present work is devoted to the construction of the effective QCD Lagrangian which is controlled not only by perturbative (asymptotically free) theory but also by non-perturbative strong coupling regime. The perturbative part of the QCD Lagrangian is described here by the renormalization group (RG) approach [1],[2].

Starting from the partition function for non-abelian SU(3)-gauge theory with quark fields Ψ,$\bar{\Psi}$ and gluon fields A^j_μ (j=1,...,8 is the color index) we have the following Euclidean functional integral:

$$Z = \frac{1}{N}\int [\mathcal{D}A][\mathcal{D}\bar{\Psi}][\mathcal{D}\Psi]e^{-S(A,\bar{\Psi},\Psi)}. \tag{1}$$

Assuming the appearance of the effective QCD action S_{eff} as a result of the integration over quark fields:

$$Z = \frac{1}{N}\int [\mathcal{D}A]e^{-S_{eff}(A)}, \tag{2}$$

we have the following effective Lagrangian in Minkowski space:

$$S_{eff} = \int d^4x \mathcal{L}_{eff}, \qquad \mathcal{L}_{eff} = -\frac{\alpha^{-1}_{eff}(F^2)}{16\pi}F^2, \tag{3}$$

where $F^2 \equiv (F^j_{\mu\nu})^2$, $\quad F^j_{\mu\nu} = \partial_\mu A^j_\nu - \partial_\nu A^j_\mu + f^{jkl}A^k_\mu A^l_\nu$, $\quad \alpha_{eff}(F^2) = g^2_{eff}/4\pi$ and g_{eff} is the effective QCD coupling constant.

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Below we consider the quantum corrections to the classical QCD action, stipulated by the vacuum polarization induced not only by quark loops, but also by gluon loops including ghosts. These contributions don't change the form of the effective Lagrangian given by Eq. (3).

The usual procedure used to estimate the generating functional:

$$Z[J] = \frac{1}{N} \int [\mathcal{D}A] e^{-S_{eff}(A) + \int J^j_\mu A^j_\mu d^4x} \tag{4}$$

is to assume that the integral over all paths is dominated by minima of the action $S_{eff}(A)$. For asymptotically free theories (including QCD) the minimum of $S_{eff}(A)$ is not the classical vacuum $F^j_{\mu\nu} = 0$. The quantum fluctuations in the strong coupling regime give rise to spontaneous magnetization of the vacuum and the non-trivial vacuum field - the gluon condensate - comes into existence: $F^2 = F_0^2 \neq 0$. Using QCD sum rules the authors of Ref.[3] have obtained the following result:

$$F_0^2 \approx 0.5 \text{ GeV}^4. \tag{5}$$

The effective action S_{eff} gives the correct trace anomaly for the energy-momentum tensor $\theta_{\mu\nu}$:

$$\theta^\mu_\mu = \frac{\beta(g)}{2g^3} F^2, \tag{6}$$

where the effective coupling constant $g_{eff} \equiv g$ is a function of the variable:

$$t = \frac{1}{2} \ln \frac{F^2}{\mu_R^4} = \ln \frac{\mu^2}{\mu_R^2}, \tag{7}$$

(μ is the energy scale and μ_R is the renormalization mass) and $\beta(g)$ is the well-known Callan-Symanzik β-function.

The effective action is RG invariant:

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta(g) \frac{\partial}{\partial g} \right] S_{eff} = 0, \tag{8}$$

and $\alpha_s = g^2(t)/4\pi$ obeys the RG equation:

$$\frac{d \ln \alpha_s(t)}{dt} = \beta(\alpha_s). \tag{9}$$

Considering the 4-loop approximation we have:

$$\beta(\alpha_s) \approx -(\beta_0 \tilde{\alpha} + \beta_1 \tilde{\alpha}^2 + \beta_2 \tilde{\alpha}^3 + \beta_3 \tilde{\alpha}^4) \tag{10}$$

where $\tilde{\alpha} \equiv \alpha_s/4\pi$, $\quad \beta_0 = 11 - \frac{2}{3} N_f$, $\quad \beta_1 = 102 - \frac{38}{3} N_f$,

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2, \qquad [4]$$

$$\beta_3 = \left(\frac{149753}{6} + 3564\zeta_3\right) - \left(\frac{1078361}{162} + \frac{6508}{27}\zeta_3\right) N_f + \left(\frac{50065}{162} + \frac{6472}{81}\zeta_3\right) N_f^2 + \frac{1093}{729} N_f^3, \qquad [5]$$

Here ζ_s is the Riemann $\zeta-$function: $\zeta_3 \approx 1.202056903...$ and N_f is the number of quarks having masses $m_q < \mu$.

The solution of Eq.(9) in the case of 3-loop approximation gives the following result:

$$\alpha^{-1}(t) \approx \alpha_R^{-1} + \frac{\beta_0}{4\pi}t + \frac{\lambda_1}{2} \ln \frac{\alpha_t^{-2} + \lambda_1\alpha_t^{-1} + \lambda_2^2}{\alpha_R^{-2} + \lambda_1\alpha_R^{-1} + \lambda_2^2} + \frac{\lambda_1^2 - 2\lambda_2^2}{\sqrt{4\lambda_2^2 - \lambda_1}} \arctan \frac{\sqrt{4\lambda_2^2 - \lambda_1^2}(1 - \alpha_R\alpha_t^{-1})}{(2 + \lambda_1\alpha_R)\alpha_t^{-1} + \lambda_1 + 2\lambda_2^2\alpha_R}, \qquad (11)$$

where we have used the following designations: $\alpha_t \equiv \alpha_s(t)$, $\alpha_R \equiv \alpha_s(\mu_R^2)$, $\lambda_1 = \frac{1}{4\pi}\frac{\beta_1}{\beta_0}$, $\lambda_2^2 = \frac{1}{(4\pi)^2}\frac{\beta_2}{\beta_0}$.

We have also performed the computer solution of RG equation (9) in the case of 4-loop approximation. Starting from the value

$$\alpha_s(M_Z) = 0.118 \pm 0.003, \qquad [6] \qquad (12)$$

using the following (experimentally given) masses of c- and b-quarks [6]:

$$m_c = 1.3 \pm 0.3 \quad \text{GeV}, \quad m_b = 4.3 \pm 0.2 \quad \text{GeV} \qquad (13)$$

and varying the number of quarks N_f from $N_f = 3$ to $N_f = 5$ we have reproduced the function $\alpha_s^{-1}(t)$ in the perturbative region of energy scale $\mu \geq 1$ GeV (see Fig.1 where the curves 1,..4 correspond to the n-loop approximations with n = 1,..4). It was not difficult to construct the effective QCD Lagrangian with the help of this behavior of $\alpha_s^{-1}(t)$.

The function $U = -\mathcal{L}_{eff}$ where $\mathcal{L}_{eff}$ is given by Eq.(3) coincides with the effective potential U_{eff} only for constant $A_\mu^j(x)$ when $F_{\mu\nu}^j = f^{jkl}A_\mu^k A_\nu^l$. The perturbative function U is presented in Fig.2.

The general idea is to construct the curve $U(F^2)$ which, according to the confinement scenario of Ref.[7], obeys the following requirements:

- $U(F^2) = 0$;
- $U(F^2)$ has the minimum at $F^2 = F_0^2$;
- the function $U(F^2)$ may be approximated by the Taylor expansion in the vicinity of the minimum:

$$U(F^2) \approx \sum_{n=0}^{3} a_n (F^2 - F_0^2)^n. \qquad (14)$$

The last approximation for $U(F^2)$ may be smootly sewn together with $U = -\mathcal{L}_{eff}$ derived from RG equation in the region $\alpha_s \sim 0.35$.

The result of such a procedure is given in Fig.3 and Table I.

Table I.

α_s at sewing point	condensate F_0^2, GeV^4			$\frac{i\text{–th loop}}{j\text{–th loop}}$, %				
	$+\sigma$	central	$-\sigma$	4/3	4/1	3/2	3/1	2/1
0.33	0.39	0.21	0.11	52%	1.7%	21%	3.4%	16%
0.35	0.28	0.15	0.08	55%	2%	22%	3.8%	17.2%
0.4	0.16	0.08	0.04	60%	4.3%	32%	7.2%	23%

The designation "central $\pm\sigma$" means:

$$\begin{pmatrix} \alpha_s \\ m_c \\ m_b \end{pmatrix} = \begin{pmatrix} 0.118 \\ 1.3 \\ 4.3 \end{pmatrix} \pm \begin{pmatrix} 0.003 \\ 0.3 \\ 0.2 \end{pmatrix} \tag{15}$$

The case "central $+\sigma$" gives the gluon condensate value close to the result of Ref.[3] : $F_0^2 \approx 0.39$ GeV^4 (compare with Eq.(5)). The corresponding curves for $U(F^2)$ and $\alpha_s^{-1}(t)$ are presented in Fig.3 and Fig.4. The function plotted in Fig.4 describes the behaviour of $\alpha_s^{-1}(t)$ not only in the perturbative, but also in the non-perturbative regions showing $\alpha_s(F_0^2 \approx 0.39 \text{ GeV}^4 \Leftrightarrow \mu \approx 0.79 \text{ GeV}) \approx -0.36$.

The equations of motion following from the effective Lagrangian (3):

$$\mathcal{D}_\lambda(\varepsilon F^i_{\lambda\mu}) = j^i_\mu \qquad (i = 1, ...8) \tag{16}$$

($\mathcal{D}_\lambda$ is a covariant derivative and j_μ is a current) contain the chromodielectric function $\varepsilon(t)$:

$$\varepsilon(t) = \frac{1}{4\pi}\left[\frac{d\alpha^{-1}}{dt} + 2\alpha^{-1}(t)\right]. \tag{17}$$

It is not difficult to construct the function $\varepsilon(t)$ with the help of the function $\alpha_s(t)$ plotted in Fig.4. The preliminary investigation of solutions of the equations of motion (16) begins to show the existence of QCD strings.

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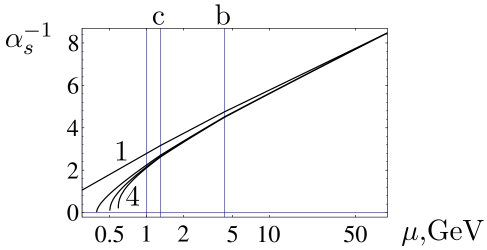

Fig.1

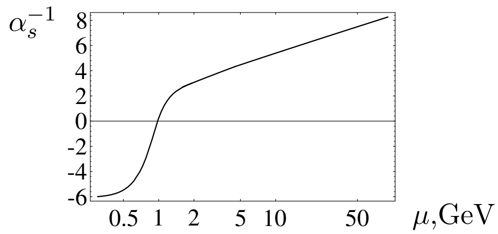

Fig.4

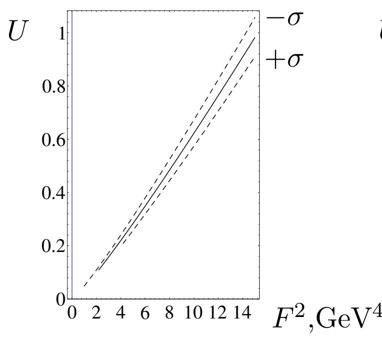

Fig.2

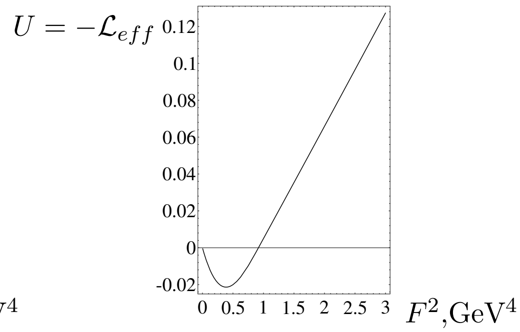

Fig.3

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